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20-2 Central Bank Policy Sets the Lower Bound on Bond Yields

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ABSTRACT

This paper shows that the scope for bond yields to fall below zero is strictly limited by market expectations about how far below zero central banks are willing to set their short-term policy rates. If a central bank communicates a credible commitment to keeping its policy rate above a given level under all circumstances, then bond yields must be higher than that level. This result holds true even in a model in which central banks are able to depress the term premium in bond yields below zero via large-scale purchases of long-term bonds, also known as quantitative easing (QE). QE becomes less effective as bond yields approach their lower bound.

JEL codes: E43, E58, G12
Keywords: Negative interest rate, quantitative easing

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INTRODUCTION

For the first time in history, long-term bonds are trading at negative yields in major economies. Investors who buy these bonds and hold them to maturity will lose money on their investment under all circumstances. This paper shows that the scope for bond yields to fall below zero is strictly limited by market expectations about how far below zero central banks are willing to set their short-term policy rates. If a central bank communicates a credible commitment to keeping its policy rate above a given level under all circumstances, then bond yields must be higher than that level. That is, the lower bound for the short-term interest rate is a lower bound for the interest rate at all maturities.

Quantitative easing (QE) lowers bond yields by lowering the expected path of future short-term rates and/or the term premium.[1] However, QE becomes less potent as bond yields approach the lower bound because there is less scope to lower the expected future path of short-term rates and because the term premium shrinks toward zero as yields approach their lower bound.

Our argument assumes rational expectations but is otherwise very general. Our point would be trivial in the expectations theory of the term structure, according to which the long-term interest rate is equal to the average of future expected short-term interest rates.[2] We do not adopt such a model, however, because it would imply that QE operations do not work as there is no risk premium. Our point is based on an arbitrage argument in a more general model in which investors are risk averse and they may have different horizons consistent with the habitat theory of the term structure.[3] The model allows for a negative term premium, including when QE is used to reduce the supply of long-term bonds available to investors. The results are robust to the presence of financial frictions limiting the leverage that investors can take to arbitrage interest rate differences.

Bond market participants in several countries appear to have been surprised when central banks in those countries lowered their policy rates below zero at various times between 2012 and 2016. Bond yields dropped between 10 and 30 basis points in the weeks surrounding these policy moves (Ubide 2017, table 4.2; Christensen 2019). Of particular note, Denmark was the first country to adopt a negative policy rate on July 5, 2012.[4] Its 10-year bond yield fell 7 basis points on that day and 37 basis points over the month of July.[5] Bond yields fell between 1 and 8 basis points in a selection of advanced economies on that day, except for

1 The term premium is the excess yield that investors require to commit to holding a long-term bond instead of shorter-term bonds. The premium may be negative if investors value the certainty of long-term bond returns more than the uncertain returns of a series of short-term bonds.

2 See Ruge-Murcia (2006) for an analysis of the impact of the zero lower bound for long-term interest rates under the pure expectations hypothesis.

3 See Vayanos and Vila (2009) for a preferred-habitat model of the term structure based on similar assumptions.

4 Switzerland imposed negative interest rates on foreigners in the 1970s to discourage upward pressure on the Swiss franc. However, Swiss interest rates were positive for domestic residents until the past few years.

5 Daily bond yields are from Bloomberg. Month-end yields are from Haver Analytics.

Canada where they rose 1 basis point.[6] Yields fell an average of 21 basis points in these countries over the month of July. Bond market participants appear to have started reassessing their assumptions concerning the lower bound on policy rates following the path-breaking Danish move in July 2012 (Grisse, Krogstrup, and Schumacher 2017).

To date, bond yields have not dropped below levels that might reasonably reflect a lower bound on short-term yields. For example, in the countries with the lowest bond yields—Germany and Switzerland—5-year and 10-year Treasury bond yields only briefly touched (in August 2019) the lowest levels observed for 3-month Treasury yields and are currently well above those levels.[7] The next section builds a model to explain this empirical finding.

A MODEL OF BOND YIELDS

The model can have an arbitrary number of periods and we assume infinite time $t = 1, 2, \ldots$. We consider zero-coupon bonds. We denote by $r_t(m)$ the interest rate on a zero-coupon bond that is issued in period t and pays off in $t + m$. The short-term interest rate in period t is $r_t(1)$ and the yield curve in period t is characterized by the interest rate $r_t(m)$ corresponding to different maturities m.[8]

We then show the following property:

(***P***) If $r_t(1) \geq \underline{r}$ for all t, then $r_t(m) \geq \underline{r}$ for all t and m.

The property can be explained as follows, starting with $m = 2$. Assume that an investor wants to invest for two periods, between t and $t + 2$ (see figure 1). The investor can buy bonds that mature in $t + 2$, which yield $[1 + r_t(2)]^2$ per dollar invested in period t. Alternatively, the investor can buy 1-period bonds and reinvest the proceeds in the next period. This yields $[1 + r_t(1)][1 + r_{t+1}(1)]$. The next-period return $r_{t+1}(1)$ is not known in period t but if the short-term interest rate cannot fall below $\underline{r}$, then the second strategy cannot yield less than $[1 + r_t(1)]$ $[1 + \underline{r}]$, which, in turn, cannot be less than $[1 + \underline{r}]^2$. For investors to hold 2-period bonds, thus, it must be that $r_t(2) \geq \underline{r}$, or else holding a sequence of short-term bonds would be more profitable under all states of nature.

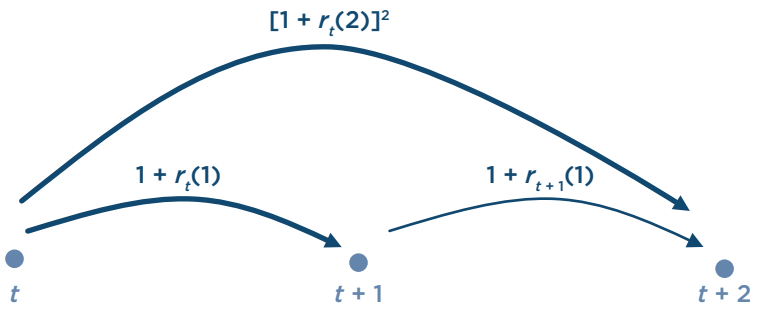
Yet another option is to invest in a 2-period bond in period t and sell it in period $t + 1$ to buy a 1-period bond. However, because a 1-year-old 2-period bond and a new 1-period bond both have fixed and known payouts, they must have the same

6 The countries are Canada, France, Germany, Japan, Sweden, Switzerland, the United Kingdom, and the United States. Italy is not included because it was hit by the euro crisis that summer.

7 Based on month-end data, the lowest observations on 3-month yields were –0.98 and –1.12 in Germany and Switzerland, respectively, in 2016. The lowest observations on 5-year yields were –0.94 and –1.16 in August 2019. 10-year yields have been consistently higher than 5-year yields over the past few years.

8 We equate the central bank's policy rate with the yield on 1-period bonds. Although this is approximately true, different choices of policy instrument may lead to small but noticeable deviations between the policy rate and the short-term bond rate. For example, the Federal Reserve's policy rate is an interbank loan rate and its primary instrument is the rate of interest paid on bank reserves. If short-term bonds (bills) have desirable properties not available in loans to banks and reserves, such as the ability to be posted as collateral or to be held by a broader range of investors, they may have lower yields than the rates set by the Fed. The lower bound we show is based on short-term bond yields and may be slightly different from the policy rate.

Figure 1
Alternative investment returns



yield for investors to freely hold them.[9] Thus, the existing 2-period bond is valued at its ultimate payout discounted by the current 1-period yield, $\frac{[1+r_t(2)]^2}{[1+r_{t+1}(1)]}$. The payout of this option is $\frac{[1+r_t(2)]^2}{1+r_{t+1}(1)} \times [1+r_{t+1}(1)]$, which is the same as holding the existing bond to maturity.

Now consider the case in which $m = 3$. Investors wishing to invest for 3 periods have four options:[10] (1) they can buy and hold a 3-period bond yielding $[1 + r_t(3)]^3$; (2) they can buy a sequence of 1-period bonds yielding $[1 + r_t(1)][1 + r_{t+1}(1)][1 + r_{t+2}(1)]$; (3) they can buy a 2-period bond and roll it over into a 1-period bond yielding $[1 + r_t(2)]^2[1 + r_{t+2}(1)]$; or (4) they can buy a 1-period bond and roll it over into a 2-period bond yielding $[1 + r_t(1)][1 + r_{t+1}(2)]^2$. By assumption, $r_t(1) \geq \underline{r}$ for all t. We showed above that $r_t(2) \geq \underline{r}$ for all t. Thus, it follows that $r_t(3) \geq \underline{r}$ for all t or no investor would hold a 3-period bond.

Similar results hold as m is increased. The appendix contains a general proof of property ***P***. The proof is not sensitive to the addition of other asset classes. It relies only on the assumption that some investors freely hold longer-term bonds.

9 Bonds of nearly the same maturity may have different returns, as shown by the literature on the "on-the-run" spread. The on-the-run spread reflects a premium paid for, and thus lower return on, bonds that are more liquid. Allowing for such a spread only strengthens our result because the current short-term bond, whose rate is set by the central bank, is more liquid and thus has a lower yield than the old long-term bond. Anticipation of this liquidity difference puts upward pressure on the new long-term interest rate relative to expected future short-term rates. In recent years the on-the-run spread has been very small. Yield differences between US Treasury securities of similar maturity are around 10 basis points or less for maturities under 1 year, falling to less than 5 basis points for maturities above 5 years (Bloomberg data for December 13, 2019). Yield differences were greater at times in the past, with the spread for near-10-year maturities reaching a peak of 60 basis points during the financial crisis of 2008 (Adrian, Fleming, and Vogt 2017, table 5).

10 They also have the option of buying a 2-period bond and selling it before maturity, but, as shown above, the payout of this option is the same as that of holding the bond to maturity. The option of selling a 3-period bond before maturity still leads to the result that the initial 3-period bond yield must be greater than or equal to the lower bound because the future 1-period and 2-period yields are also subject to that bound.

Conceivably, the central bank can push yields below $\underline{r}$ if it buys all long-term bonds in the market plus the maximum amount that can be shorted by investors.[11] But this would take QE out of the realm of open-market operations and lead to a disconnect between the long-term bond market and the markets for other financial assets.[12] The same outcome could be achieved by having the central bank lend on below-market terms to selected borrowers. Bond yields below $\underline{r}$ would have a stimulative effect only to the extent that the central bank's inevitable losses constitute a fiscal transfer to investors.

NUMERICAL EXAMPLES

This section calculates bond yields and term premiums for a simplified 3-period version of the model in the appendix with two classes of investors, each endowed with 1 unit of income to invest in period 0. The first class of investors is short-lived and cares only about consumption in period 1. They are replaced by a new group of short-lived investors in period 1, also endowed with 1 unit of income, who care only about consumption in period 2. The second class of investors is long-lived and cares only about consumption in period 2.

Utility of each class of investors is defined by constant relative risk aversion in consumption above an assumed subsistence level of 0.9: $U = \frac{(C-0.9)^{1-\gamma}}{1-\gamma}$. In period 0, both short-lived and long-lived investors allocate their endowments between 1-period bonds and 2-period bonds to maximize expected utility. In period 1, returns on both assets are known in advance and must be equal if both are freely held. Thus, investors in period 1 have no meaningful investment decision.

The central bank sets the short-term interest rate $r_0(1)=0$ in period 0. Investors must form expectations about the short-term interest that will be set in period 1. It is assumed to follow a simple binomial distribution in which it takes the value r_L with probability p and r_H with probability $1–p$. The government issues both short-term and long-term bonds and sets total bond supply equal to the combined income of both classes of investors.[13] This is not a complete model of the economy. There are assumed to be other sectors, such as firms and workers, who may produce goods, pay taxes, and receive transfers. Returns on government bonds (which may be positive or negative) are assumed to be financed by these other sectors. We do not need to model the other sectors because they have no impact on investors.

Consumption of short-lived investors (in period 1) is

$$C1 = b1\big(1 + r_0(1)\big) + \frac{(1-b1)(1+r_0(2))^2}{(1+r_1(1))}, \quad (1)$$

11 Because holding short-term bonds would yield more than holding long-term bonds under all states of nature, investors would have an unbounded appetite for short-term bonds funded by short positions in long-term bonds. However, margin requirements limit the amount that investors can short long-term bonds. Moreover, if the central bank holds all long-term bonds, it could choose not to lend them, making it impossible for investors to take short positions in long-term bonds.

12 This disconnect would also occur if the only remaining private holders of long-term bonds are institutions required to hold them by regulation. In the event of a disconnect, such a regulation would be difficult to justify as it would impose unconditional losses on the affected institutions.

13 The addition of other assets would change these results depending on the covariance of asset returns. But the general result that the long-term rate cannot be lower than the lower bound on the short-term rate is not sensitive to adding more assets.

where $b1$ denotes the share of endowment invested in short-term bonds at interest rate $r_0(1)$. The remainder of the endowment, $1–b1$, is invested in long-term bonds that must be sold in period 1 to finance consumption. Their value equals their period 2 payoff, $[1 + r_0(2)]^2$, discounted by the short-term interest rate in period 1. Consumption of long-lived investors (in period 2) is

$$C2 = b2\big(1 + r_0(1)\big)\big(1 + r_1(1)\big) + (1 - b2)(1 + r_0(2))^2, \quad (2)$$

where $b2$ denotes the share of endowment invested in short-term bonds analogous to $b1$ above. The first-order conditions for maximization of utility with respect to the allocation of endowment across short-term and long-term bonds are

$$E\left\{(C1 - 0.9)^{-\gamma}\left[\big(1 + r_0(1)\big) - \frac{\big(1+r_0(2)\big)^2}{\big(1+r_1(1)\big)}\right]\right\} = 0 \text{ and} \quad (3)$$

$$E\left\{(C2 - 0.9)^{-\gamma}\left[\big(1 + r_0(1)\big)\big(1 + r_1(1)\big) - \big(1 + r_0(2)\big)^2\right]\right\} = 0, \quad (4)$$

where $E\{\}$ denotes the expectations operator with respect to the random variable $r_1(1)$. Finally, the sum of the investment shares across investor classes must equal the exogenously supplied amount of short-term bonds, expressed as a share of the total bond supply of 2.

$$b1 + b2 = (Short\text{-}term\ Share\ of\ Issuance) \times 2 \quad (5)$$

For given values of the parameters and exogenous variables, equations (1) to (5) can be solved to find the long-term bond yield in period 0, $r_0(2)$, and the investment shares, $b1$ and $b2$.

Table 1 displays the long-term bond yield and the investment shares under various sets of parameters when no restrictions are imposed on the investment shares, so that some investors may take negative, or short, positions in some bonds. (Table 2 considers the case in which short positions are not allowed.) All scenarios assume that γ=3, p=0.5, and the initial short-term interest rate is 0. The first four rows of table 1 display the case in which the future short-term interest rate has equal probabilities of being either 0 or 4. In the absence of any term premium, the long-term bond yield would be 1, which is the average of the current short-term yield of 0 and the expected future short-term yield of 2; this notional expectations yield is denoted by $r_0(2)^{exp}$ on the left side of the table. When the short-term share of issuance is 0.50, the term premium is 0 and the actual long-term bond yield is 0.99 (the middle column). Short-lived investors hold 98 percent of their wealth in short-term bonds and long-lived investors hold 98 percent of their wealth in long-term bonds.[14] Both groups are almost completely hedged from consumption risk.

Differences across the columns display the implications of progressively raising the share of short-term bonds from 0 in the leftmost column to 1 in the rightmost column. Moving from left to right is equivalent to conducting QE bond purchases, which reduce the supply of long-term bonds and increase the

14 Short-lived investors would like to invest a small share of their endowment in long-term bonds with an expected return very slightly above 0. This bids down the bond yield a tiny amount, which induces long-lived investors to hold an equally small share of short-term bonds.

Table 1
Long-term bond yields and investment shares under different scenarios: Unconstrained investors, γ=3, $r_0(1)$=0, p=0.5

Future interest rate bounds	Variables solved	Short-term share of issuance				
		0.00	0.25	0.50	0.75	1.00
r_L=0	$r_0(2)$	1.24	1.12	0.99	0.85	0.72
r_H=4	Term premium	0.25	0.13	0.00	−0.14	−0.27
$r_0(2)^{exp}$=1	$b1$	0.53	0.75	0.98	1.21	1.45
	$b2$	−0.53	−0.25	0.02	0.29	0.55
r_L=0	$r_0(2)$	0.57	0.53	0.50	0.46	0.43
r_H=2	Term premium	0.07	0.03	0.00	−0.04	−0.07
$r_0(2)^{exp}$=0.5	$b1$	0.51	0.74	0.98	1.22	1.46
	$b2$	−0.51	−0.24	0.02	0.28	0.54
r_L=−1	$r_0(2)$	0.75	0.63	0.49	0.35	0.21
r_H=3	Term premium	0.26	0.13	0.00	−0.15	−0.28
$r_0(2)^{exp}$=0.5	$b1$	0.52	0.75	0.98	1.23	1.48
	$b2$	−0.52	−0.25	0.02	0.27	0.52
r_L=−1	$r_0(2)$	0.07	0.03	0.00	−0.04	−0.08
r_H=1	Term premium	0.07	0.04	0.00	−0.04	−0.08
$r_0(2)^{exp}$=0	$b1$	0.49	0.73	0.98	1.23	1.49
	$b2$	−0.49	−0.23	0.02	0.27	0.51

γ = risk aversion parameter in utility function; p = probability of future short-term interest rate at lower bound; r_L = lower bound; r_H = upper bound; $r_0(1)$ = initial short-term interest rate; $r_0(2)$ = initial long-term bond yield; $b1$, $b2$ = investment shares

Source: Authors' calculations based on equations (1) to (5) and various parameter values using the Solver add-in in Excel.

supply of short-term bonds. In the columns to the right, the short-term bond share rises and long-term bonds become scarce; long-lived investors are not able to fully hedge their consumption risk. The term premium becomes negative because these investors are willing to forego some expected return in short-term bonds for the safety of long-term bonds. Short-lived investors buy more than their entire endowment in short-term bonds ($b1>1$) and finance this by shorting

the long-term bond. This enables long-lived investors to buy more long-term bonds than are issued by the government. In the extreme case in which the government issues no long-term bonds (the final column), long-lived investors lend 45 percent of their endowments to short-lived investors and the long-term bond yield is well below the expected return on short-term bonds with a term premium of –0.27.

In the columns to the left, the short-term share falls and long-term bonds become plentiful; short-lived investors are not able to fully hedge their consumption risk. The term premium becomes positive because these investors are willing to forego some expected return in long-term bonds for the safety of short-term bonds. Short-lived investors are induced to lend as much as 53 percent of their endowments to long-lived investors ($b1=0.53$) when all issuance is in long-term bonds and short-term bonds have 0 net supply (the first column). Long-lived investors have a short position in short-term bonds ($b2=-0.53$, a liability to short-lived investors), which they use to finance purchases of additional long-term bonds. In this extreme case, the long-term bond yield is well above the expected return on short-term bonds and the term premium is 0.25.

The next four rows show that a reduction in the expected future short-term interest rate via a reduction in its upper bound reduces long-term interest rates and shrinks term premiums toward zero because of reduced risk. In the middle column, with no term premium, the reduction in the expected returns to holding short-term bonds shows up fully in the long-term yield. However, in the columns to the right, the decline in the long-term yield is reduced because the term premium increases (becomes less negative). In the columns to the left, the decline in the long-term yield is increased because the term premium falls.

The following four rows show the implications of reducing the lower bound on the future short-term rate simultaneously with an increase in the upper bound that leaves the expected value unchanged. In the middle column with no term premium, the long-term bond yield is essentially unchanged.[15] However, in the columns to the right with a negative term premium, this increased uncertainty over the future short-term rate reduces long-term yields because it makes the term premium more negative. The intuition is that the risk of a negative future short-term interest rate makes long-term bonds more attractive to long-lived investors and this makes the term premium more negative when long-term bonds are scarce.

The final four rows show the implications of setting the upper and lower bounds on the future short-term rate symmetric around 0, at 1 percent and –1 percent, respectively. The term premiums are relatively small because the risk on the future return is relatively small. Nevertheless, the bond yield is negative when long-term bonds are scarce, reflecting the risk of a negative future short rate.

For given parameters, QE (increasing the short-term share of bond supply) always makes the term premium more negative, but it can never push the long-term yield below the lower bound on the short-term rate. Greater risk aversion also makes the term premium more negative when long-term bonds are scarce, but no amount of risk aversion can push the bond yield below the lower bound on the short rate.

15 Tiny deviations arise because of Jensen's inequality.

Table 2
Long-term bond yields and investment shares under different scenarios: Constrained investors, γ=3, $r_0(1)$=0, p=0.5

Future interest rate bounds	Variables solved	Short-term share of issuance				
		0.00	0.25	0.50	0.75	1.00
r_L=0	$r_0(2)$	1.47	1.25	0.99	0.75	0.53
r_H=4	Term premium	0.48	0.26	0.00	−0.24	−0.46
$r_0(2)^{exp}$=1	$b1$	0.00	0.50	0.98	1.00	1.00
	$b2$	0.00	0.00	0.02	0.50	1.00
r_L=0	$r_0(2)$	0.63	0.57	0.50	0.43	0.37
r_H=2	Term premium	0.14	0.07	0.00	−0.07	−0.13
$r_0(2)^{exp}$=0.5	$b1$	0.00	0.50	0.98	1.00	1.00
	$b2$	0.00	0.00	0.02	0.50	1.00
r_L=−1	$r_0(2)$	0.98	0.76	0.49	0.23	0.00
r_H=3	Term premium	0.49	0.27	0.00	−0.27	−0.50
$r_0(2)^{exp}$=0.5	$b1$	0.00	0.50	0.98	1.00	1.00
	$b2$	0.00	0.00	0.02	0.50	1.00
r_L=−1	$r_0(2)$	0.14	0.07	0.00	−0.08	−0.15
r_H=1	Term premium	0.14	0.07	0.00	−0.07	−0.14
$r_0(2)^{exp}$=0	$b1$	0.00	0.50	0.98	1.00	1.00
	$b2$	0.00	0.00	0.02	0.50	1.00

γ = risk aversion parameter in utility function; p = probability of future short-term interest rate at lower bound; r_L = lower bound; r_H = upper bound; $r_0(1)$ = initial short-term interest rate; $r_0(2)$ = initial long-term bond yield; $b1$, $b2$ = investment shares

Source: Authors' calculations based on equations (1) to (5) and various parameter values using the Solver add-in in Excel. The solution is subject to the restrictions $0 \le b1, b2 \le 1$. Equation (3) is dropped when the short-term share of issuance exceeds 50 percent and $b1$=1. Equation (4) is dropped when the short-term share is less than 50 percent and $b2$=0.

Table 2 shows results in which investors are not allowed to take negative positions ($0 \le b1, b2 \le 1$). Investors at a corner solution (long-lived in the left columns and short-lived in the right columns) are assumed to have no impact on the long-term yield. The yield is determined to satisfy the first-order conditions only of those investors who hold both types of bonds. In the extreme case of 0

short-term bonds, the solution is based on short-term investors and vice versa for the extreme case of 0 long-term bonds. The term premiums are uniformly larger. Other results are unchanged. In particular, QE and higher risk aversion can never push the long-term yield below the lower bound on the short rate. Negative long-term yields are possible only when the lower bound, r_L, is negative.

CONCLUSION

This paper shows that the lower bound for the short-term interest rate is a lower bound for the interest rate at all maturities. This result places a limit on how far bond yields can be reduced via quantitative easing. As bond yields approach the lower bound there is less scope to lower the expected future path of short-term rates and the term premium shrinks toward zero.

An important implication of these results is that the impact of QE on bond yields (or the term premium) is not determined solely by the amount of bonds purchased, as has been assumed in some studies (Kiley and Roberts 2017, Kiley 2018, Chung et al. 2019). In his presidential address to the American Economic Association, Ben Bernanke (2020) used techniques similar to these studies to explore the scope for QE to deliver monetary stimulus when short-term interest rates are constrained near zero. He noted that when the equilibrium real interest rate is very low, model simulations of aggressive QE policies imply that bond yields are frequently below the zero bound on short-term rates. He cautioned that such simulations "may overstate the amount of stimulus that is attainable in practice." The results of this paper support Bernanke's warning and the problem may be greater than implied by simulated bond yields below zero. As interest rates approach a perceived lower bound, QE bond purchases will have a smaller impact on bond yields even before they reach that bound, suggesting that existing models may understate the amount of QE purchases needed just to approach zero.

Another implication of these results is that the additional policy space created by successfully raising the inflation target is much greater than previously believed because higher inflation increases the scope to reduce bond yields via forward guidance and QE (Gagnon and Collins 2019).

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APPENDIX
PROOF IN A GENERAL MODEL

Property (***P***) can be proved under very general assumptions about the composition and preferences of investors. Individual investors could have short or long horizons, they could value income in every period or in a single period, and the composition of investors could change over time.

Assumptions

The economy is populated by a large number of investors indexed by i. To fix the ideas, we assume that the period-t utility of investor i is given by

$$U_t^i = E_t\left[\sum_{s=0}^{+\infty} \delta_{t+s}^i \beta^s u\left(c_{t+s}^i\right)\right] \quad \text{(A1)}$$

where δ_t^i is an indicator variable for whether investor i consumes in period t ($\delta_t^i = 1$) or not ($\delta_t^i = 0$). We assume, to alleviate notations, that investors have the same discount factor β and the same utility function $u(\cdot)$ but this does not matter for our results.

Investors could enter or exit the bond market. An investor who exits the bond market in period t has $\delta_{t+s}^i = 0$ for all $s > 0$.

As in Vayanos and Vila (2009), investors could have short or long horizons. In period t, an investor with a 1-period horizon is an investor who consumes in period $t+1$ and then exits the bond market. More generally, an investor with horizon m is an investor who consumes only in period $t + m$. Even more generally, investors could have a stochastic δ and their horizon could be measured by the likelihood that they consume in the near vs. late future.

The composition of investors could change over time. As investors leave the market, some new investors with different preferences might come in and change the average horizon of investors in the market.

In order to write the investors' budget constraints, it will be convenient to use the market prices of zero-coupon bonds. We denote by $q_t(m)$ the period-t price of a zero-coupon bond that pays one unit in period $t + m$. It is related to the per-period interest rate through

$$q_t(m) = [1 + r_t(m)]^{-m}. \quad \text{(A2)}$$

We denote by w_t^i the resources of investor i at the beginning of period t. The period-t resources are allocated to consumption or reinvestment

$$w_t^i = c_t^i + \sum_{m=1}^{M} q_t(m) b_t^i(m), \quad \text{(A3)}$$

where $b_t^i(m)$ is the quantity of zero-coupon bonds of maturity m purchased by the investor, and M is the longest maturity available in the market.

For a new entrant w_t^i is an exogenous endowment. For an investor who was previously in the market this results from his previous portfolio allocation,

$$w_t^i = \sum_{m=1}^{M} q_t(m-1) b_{t-1}^i(m) \quad \text{(A4)}$$

with $q_t(0) = 1$. Investors are subject to borrowing constraints

$$b_t^i(m) \geq \underline{b}_t^i(m), \quad \text{(A5)}$$

where the exogenous limits $\underline{b}_t^i(m)$ are nonpositive. The borrowing limits could be determined by the agent's income or intertemporal wealth and in this sense could be interpreted as leverage constraints. We do not consider the case where the limits are positive, i.e., the investors are constrained to hold a minimum amount of bonds. (Property ***(P)*** is not necessarily true if investors are constrained to hold a minimum amount of long-term bonds.)

Finally, we assume exogenous positive supplies of bonds (the model would not be difficult to extend to the case where supply is price-elastic). Denoting with an uppercase letter the total supply of zero-coupon bonds of any given maturity m we have

$$\sum_i b_t^i(m) = B_t(m) \tag{A6}$$

for all $m \leq M$, with $B_t(m) > 0$.

Proof of property (P)

We prove the property by induction. We are going to show that if *(P)* is true for maturity m, then it is true for maturity $m + 1 \leq M$. Since the property is true (by assumption) for $m = 1$, it will follow that it is true for any maturity $m \leq M$.

The proof comes from the inequality

$$q_t(m+1) \leq q_t(1) max_t q_{t+1}(m), \tag{A7}$$

where $max_t q_{t+1}(m)$ is the maximum possible value of $q_{t+1}(m)$ conditional on time-t information.

Inequality (A7) is formally proven below but it is intuitive. The two sides of the equation correspond to two different ways that an investor can secure at time t a unitary payment at time $t + m + 1$.

First, the investor can buy a zero-coupon bond with maturity $m + 1$. The cost of this option is the left-hand side of equation (A7). Second, the investor can invest in a short-term bond and then reinvest the proceeds in a bond of maturity m in the following period. The cost of this strategy, $q_t(1)q_{t+1}(m)$, is not known in period t, but it cannot be larger than the right-hand side of equation (A7). If condition (A7) were violated, no investor would hold bonds of maturity $m + 1$ in period t since the second strategy would yield the same payoff as the first one at a strictly lower cost. Since there is a positive supply of bonds of maturity $m + 1$, this cannot be true in equilibrium.

Using (A2), assuming that ***(P)*** is true for maturity m is equivalent to assuming

$q_t(m) \leq (1 + \underline{r})^{-m}$.

Then it follows from this equation and (A7) that

$q_t(m + 1) \leq (1 + \underline{r})^{-m-1}$,

which means that property ***(P)*** is true for maturity $m + 1$.

We prove (A7). The Bellman equation for an investor i who stays in the market is

$$V_t^i\left(w_t^i\right) = max\left\{\delta_t^i u\left(c_t^i\right) + \lambda_t^i\left[w_t^i - c_t^i - \sum_{m=1}^{M} q_t(m) b_t^i(m)\right] + \sum_{m=1}^{M} \mu_t^i(m)\left[b_t^i(m) - \underline{b}_t^i(m)\right] + E_t V_{t+1}^i\left(w_{t+1}^i\right)\right\}$$

where the maximization is over c_t^i and $b_t^i(m)$ for $m = 1, \ldots M$. Variables λ_t^i and $\mu_t^i(m)$ are the shadow costs of constraints (A3) and (A5), respectively, and w_{t+1}^i is given by (A4).

The problem admits the first-order conditions

$$\lambda_t^i q_t(m) = \beta E_t\left[\lambda_{t+1}^i q_{t+1}(m-1)\right] + \mu_t^i(m) \tag{A8}$$

for all $m \leq M$.

If investor i holds bonds of maturity $m + 1$, equation (A8) must hold for $m + 1$ with $\mu_t^i(m+1) = 0$,

$$\lambda_t^i q_t(m+1) = \beta E_t\left[\lambda_{t+1}^i q_{t+1}(m)\right].$$

Dividing this equation by equation (A8) with $m = 1$ implies

$$q_t(m+1) \leq q_t(1)\frac{E_t\left[\lambda_{t+1}^i q_{t+1}(m)\right]}{E_t\left[\lambda_{t+1}^i\right]} \leq q_t(1)\frac{E_t\left[\lambda_{t+1}^i max_t q_{t+1}(m)\right]}{E_t\left[\lambda_{t+1}^i\right]} = q_t(1) max_t q_{t+1}(m),$$

which gives (A7). For the last equality we have used the fact that $max_t q_{t+1}(m)$ being nonstochastic viewed from period t, can be taken out of the expectation operator.

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