Appendix

Latent-Variable Estimation of Japanese Potential GDP[1]

There are several methods available to measure a national economy's potential GDP. Any of these, traced out over time, gives an estimate of the long-run sustainable rate of growth of the economy, that is, the rate at which the economy can expand in line with its productivity improvements and its accumulation of the factors of production (physical and human capital). Because potential growth is by definition unobservable directly, some means must be chosen to filter out the noise of year-to-year swings in output around potential and to identify when the potential shifts. The measure referred to in this paper as the ''latent variable'' estimate of Japanese potential output follows Kuttner's (1992, 1994) approach to US data and uses a time-series technique known as a Kalman filter to identify the underlying potential growth rate in the measured GDP and inflation data.

This latent-variable approach does not rely on the analysis of the likely paths of factors of production (as the OECD's approach does), nor does it project a growth rate based on the estimated effects of various determinants of cross-national differences in growth rates (as the literature sum-

1. I am grateful to Ken Kuttner for his sharing of programs and data, as well as his advice in the preparation of this appendix.

marized in Barro [1997] does). Instead, it takes as a starting point the idea that when the growth rate is at potential, and no positive or negative output gaps have accumulated, the rate of inflation should converge to and then remain steady at some long-run "normal" level.[2] Thus, it works from limited assumptions about how GDP and prices should move and otherwise lets the data tell us the path of potential output. This method has the advantage of allowing the growth rate of potential to vary gradually in response to output and inflation data, which should incorporate changes more promptly than procedures that rely on occasional revisions of underlying data or assumption. If the rates of actual growth in GDP and in prices change rapidly, however, particularly at the beginning or end of the data sample, they may overstate the amount of the shift in potential. That is why it is one of three different methods employed in this book. It is used to give a reliable lower bound on the rate of potential growth and cumulative output gap in 1990s Japan.

Specifying the exact implementation of this technique used here, the movement of output and prices in the economy is modeled as follows. In notation, x is the natural log of real GDP, z is the output gap, x^* is potential output, and π is inflation. The following must hold in this specification:

$$\Delta x_t^* = \mu_t \qquad \text{(equation A.1)}$$

Potential output follows a random walk with time-varying drift.

$$\Delta \mu_t = \eta_t \qquad \text{(equation A.2)}$$

The drift varies as a random walk with shock.

$$z_t = \phi_1 z_{tB1} + u_t \qquad \text{(equation A.3)}$$

The output gap follows an AR(1) process, so the output gap is affected by the previous period's output gap.

$$x_t = x_t^* + z_t \qquad \text{(equation A.4)}$$

Actual observed GDP is the sum of potential output and the gap.

$$\pi_t = \pi_0 + \beta z_{tB1} + \gamma \Delta z_{tB1} + \theta_1 \pi_{tB1} + \theta_2 \pi_{tB2} + v_t \qquad \text{(equation A.5)}$$

Inflation depends on the gap, the change in the gap, and two lags of inflation itself.

In this specification: (1) while the rate of change in potential output (μ) is allowed to vary over time, there is no direct shock to the level of

2. Formally speaking, inflation is mean-reverting in this sample in the absence of aggregate demand pressures.

Table A.1 Estimated coefficients from quarterly data on Japanese inflation and output, first quarter 1971 through third quarter 1997

Parameter	Estimate	Standard error	Estimate/ standard error	Probability	Gradient
sigma u	3.4861	0.2485	14.029	0.0000	−0.0000
phi 1	0.9456	0.0344	27.513	0.0000	0.0000
sigma v	2.5478	0.1943	13.114	0.0000	−0.0000
theta 1	−0.0656	0.1505	−0.436	0.3316	−0.0000
theta 2	0.4814	0.0738	6.522	0.0000	−0.0000
gamma	−0.1088	0.1436	−0.758	0.2242	0.0000
sigma uv	−1.2109	0.2730	−4.435	0.0000	−0.0000
beta	0.2463	0.0771	3.196	0.0007	0.0000

potential output included in this model, and (2) the inflation equation is specified in terms of the level of inflation (π), not the change in inflation. These aspects differ from the implementation of this approach on US data in Kuttner (1994), as do some lagged elements of equation A.5, in order to improve the fit to Japanese inflation data, which tended to be mean reverting over the sample.

Table A.1 shows the estimated coefficients that this system of equations yields from quarterly data on Japanese inflation and output from first quarter 1971 through third quarter 1997: note the highly significant estimate of β, that is, the coefficient on the output-gap variable in the inflation equation. This is an indication that the assumed relationship between output gaps and inflation levels used to identify movements in potential output fits the Japanese data very well in this period.[3]

In estimating the model, the condition was arbitrarily imposed that $\pi_0 = 0.5$. This says that if the output gap, its change, and inflation shocks are all zero, the inflation rate will tend toward $0.5/(1 - \beta\theta_1\beta\theta_2)$, or about 4 percent at an annual rate (this is what it means for a series to be "mean-reverting"). In an "accelerationist" version, potential output is naturally defined as the level of GDP consistent with no change in inflation. In the nonaccelerationist specification used here, potential output is defined as the level of GDP consistent with inflation at some "normal" level (one could, just as arbitrarily, have picked a π_0 that defined potential as consistent with zero inflation, but that would have been more of a mismatch with long-run Japanese experience). In any event, the choice of π_0 will affect only the estimates of the *level* of potential, not of its *growth rate*

3. Using a different econometric specification, Coe and McDermott (1997) also get the result that an output gap model predicts the co-movement of inflation and output very well in Japan and throughout Asia.

(because it affects every period's potential GDP levels the same way), and it is the growth rate of potential GDP in which we are interested.

The standard deviation of the η shock must also be chosen—in this estimation, it is set equal to 0.001. Because this is quarterly data, that corresponds to a shock with a standard deviation of 0.1 percent hitting the quarterly growth rate. This parameter determines the smoothness of the potential GDP series, where a smaller standard deviation implies a smoother development of potential over time (If the standard deviation of the η shock is set to zero, potential GDP is estimated as a straight line, and the potential growth rate is invariant). All time-series based techniques for the filtering of GDP data (e.g., the Hodrik-Prescott filter) require a similar choice of smoothness parameter. As can be seen from figure 1.3, the choice of 0.001 eliminates potential GDP displaying any sort of cyclical variation with the normal business cycles, but it is set a sufficient amount over zero to generate a big kink in potential GDP in Japan in the early 1990s.

Finally, as stated previously, the potential output growth estimated for 1990s Japan by this latent variable method probably *understates* the true level and, thus, the cumulative output gap as well. To make this estimation, the relationship between the output gap and inflation (the coefficients in equation A.5 above) is assumed to be fixed throughout the entire data sample, one which extends back to the higher inflation periods of the 1970s and 1980s. Thus, without significant measured deflation, that is, negative inflation, this method cannot easily generate large (negative) output gaps. As seen in figure 1.2, however, there is reason to believe that some deflation is afoot in the Japanese economy of late, even if CPI inflation does not capture it and the April 1997 tax rise clouds the picture.

Moreover, most macroeconomists believe that in general there is some nominal rigidity of wages and prices, especially near zero inflation. In other words, they believe that changes in wages and prices will remain zero or positive for some time even when subject to forces (such as declining output and rising slack) that at a higher initial level of inflation would lead to wage and price declines. Thus, the existence of a floor on price changes at zero inflation forces this latent-variable technique to lower the size of its output-gap estimate. The approximately 3 percent decline in estimated potential-growth rate generated by this method in less than four years (1988-92) is extraordinarily large by international and historical standards and is difficult to explain except as a statistical artifact of this rigidity. Of course, even on the basis of this lower-bound estimate (worst-reasonable-case scenario) for the potential output growth rate, Japan has built up a sizable cumulative output gap, as noted in chapter 1.